

## Magnetic Field

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## Magnetic Force - Charged Particle

Moving charges create an electric current which generates a magnetic field.
$>$ The magnetic force due to a moving charged particles in a uniform magnetic field is given as

$$
\vec{F}=q(\vec{v} \times \vec{B})
$$

$>$ This equation indicates that the direction of magnetic force is always normal to both velocity and magnetic field.
$>$ The SI unit of magnetic field is Tesla ( T ) where


$$
1 \mathrm{~T}=1 \mathrm{~N} . \mathrm{s} /(\mathrm{C} . \mathrm{m})=1 \mathrm{~N} /(\mathrm{A} . \mathrm{m})
$$

## Magnetic Force - Current

>The magnetic force due to a current passing through a wire of length L is

$$
\vec{F}=i(\vec{L} \times \vec{B})
$$



## Charged Particle Circulating in Magnetic Field

$>$ If a charged particle $q$ of mass $m$ and speed v normally enters a uniform magnetic field of $B$, the path of the particle will be a circle of radius $R$. The value of the radius is

$$
R=\frac{m v}{q B}
$$

$>$ The frequency of the motion, reciprocal of the period, is

$$
f=\frac{q B}{2 \pi m} \quad \rightarrow \quad \tau=\frac{2 \pi m}{q B}
$$

$>$ The angular frequency is

$$
\omega=2 \pi f=\frac{q B}{m}
$$

## EQUAL Electric and Magnetic Forces

$>$ If a charged particle enters a region of electric and magnetic forces, the particle will move in a straight line when the net force is zero. That means the electric force equals and opposes the magnetic force. The speed of the particle must be

$$
v=\frac{E}{B}
$$

This equation is known as velocity selector.

(a)

(b)

## MAGNETIC DIPOLE - TORQUE

$>$ The magnetic dipole of a wire (coil) of N turns, area A , and current i , is defined as

$$
\mu=N i A
$$

The SI unit of magnetic dipole is A.m².
$>$ If one places the above coil in a uniform magnetic field, a torque will be experienced as

$$
\vec{\tau}=\vec{\mu} \times \vec{B}
$$

$>$ The orientation energy (potential energy) of the magnetic dipole is

$$
U=-\vec{\mu} \cdot \vec{B}
$$

Please revisit chapter 2 and compare these quantities with those of electric field.

## Worked Exercises

1. An electron of a speed of $2 \times 10^{5} \mathrm{~m} / \mathrm{s}$ perpendicularly enters a uniform magnetic field of 4 mT . Calculate (i) the magnetic force and (ii) the radius of its path.

## Solution

(i) The magnitude of the magnetic force, since the velocity is normal to magnetic field, is

$$
F=q v B=1.6 \times 10^{-19} \times 2 \times 10^{5} \times 4 \times 10^{-3}=1.28 \times 10^{-16} N
$$

(ii) The radius of its circular path is

$$
R=\frac{m v}{q B}=\frac{9.11 \times 10^{-31} \times 2 \times 10^{5}}{1.6 \times 10^{-19} \times 4 \times 10^{-3}}=2.85 \times 10^{-4} \mathrm{~m}
$$

## Worked Exercises

2. In a certain electric motor wires that carry a current of 8 A are perpendicular to a magnetic field of 50 mT . Calculate the magnetic force on each centimeter of these wires.

Solution
The magnitude of the magnetic force, since the magnetic field is normal to the wire, is

$$
F=i L B=8 \times 0.01 \times 50 \times 10^{-3}=0.004 N
$$

## Worked Exercises

3. A proton moving with a speed of $4 \times 10^{6} \mathrm{~m} / \mathrm{s}$ through a magnetic field of 4 T experiences a magnetic force of $12.8 \times 10^{-13} \mathrm{~N}$. Find the angle between the proton's velocity and the magnetic field.

## Solution

The magnitude of the magnetic force is defined as

$$
F=q v B \sin \theta
$$

Hence the angle is

$$
\sin \theta=\frac{F}{q v B}=\frac{12.8 \times 10^{-13}}{1.6 \times 10^{-19} \times 4 \times 10^{6} \times 4}=0.5 \quad \rightarrow \quad \theta=30^{\circ}
$$

## Worked Exercises

4. A proton moves with a speed of $4 \times 10^{6} \mathrm{~m} / \mathrm{s}$ normally to a magnetic field of 2.4 T . Calculate the acceleration of the proton due to the magnetic force.

## Solution

The magnitude of the magnetic force is

$$
F=q v B=1.6 \times 10^{-19} \times 4 \times 10^{6} \times 2.4=1.54 \times 10^{-12} N
$$

Hence the magnitude of its acceleration (from Newton's second law) is

$$
F=m a \quad \rightarrow \quad a=\frac{F}{m}=\frac{1.54 \times 10^{-12}}{1.67 \times 10^{-27}}=9.2 \times 10^{14} \mathrm{~m} / \mathrm{s}^{2}
$$

## Worked Exercises

5. An electron is in equilibrium under the influence of magnetic and electric forces. If the magnitude of the magnetic field is 1.2 T and its speed is $5 \times 10^{4} \mathrm{~m} / \mathrm{s}$, calculate the magnitude of the electric field.
Solution

The magnitude of the electric field is

$$
v=\frac{E}{B} \quad \rightarrow \quad E=v B=5 \times 10^{4} \times 1.2=6 \times 10^{4} \mathrm{~V} / \mathrm{m}
$$

## Worked Exercises

6. A coil of cross sectional area of $4 \times 10^{-6} \mathrm{~m}^{2}$ carries a current of 0.2 A . If the magnitude of the magnetic dipole is $12 \times 10^{-5} \mathrm{~A} . \mathrm{m}^{2}$, calculate the number of the coil's turns.

> Solution

The magnitude of the magnetic moment is

$$
\mu=N i A \quad \rightarrow \quad N=\frac{\mu}{i A}=\frac{12 \times 10^{-5}}{0.2 \times 4 \times 10^{-6}}=150 \text { turns }
$$

## Worked Exercises

7. A coil of cross-sectional area of $4 \times 10^{-6} \mathrm{~m}^{2}$ carries a current of 0.2 A . Its magnetic moment is initially aligned with an external magnetic field of 1.2 T. If the work required to rotate the dipole to $90^{\circ}$ is $6 \times 10^{-3} \mathrm{~J}$, calculate the number of the turns of the coil.

## Solution

The magnitude of the magnetic moment is

$$
\mu=N i A \quad \rightarrow \quad N=\frac{\mu}{i A}
$$

But we know that the work is

$$
W=-\Delta U=\mu B \quad \rightarrow \quad \mu=\frac{W}{B}=\frac{6 \times 10^{-3}}{1.2}=0.005 \mathrm{~A} \cdot \mathrm{~m}^{2}
$$

Hence the number of turns is

$$
N=\frac{0.005}{0.2 \times 4 \times 10^{-6}}=6250 \mathrm{turns}
$$

## Worked Exercises

8. A loop of 40 turns of cross-sectional area of $2 \times 10^{-4} \mathrm{~m}^{2}$ carries a current of 200 mA . The coil is placed in a uniform magnetic field. If the maximum torque exerted on the loop is $4 \times 10^{-4}$ N.m, calculate the magnitude of the magnetic field.

## Solution

The magnitude of the magnetic moment is

$$
\mu=N i A=40 \times 2 \times 10^{-4} \times 0.2=0.0016 \text { A. } \mathrm{m}^{2}
$$

But we know that the maximum torque is

$$
\tau=\mu B
$$

Hence the magnitude of the magnetic field is

$$
B=\frac{\tau}{\mu}=\frac{4 \times 10^{-4}}{0.0016}=0.25 \mathrm{~T}
$$

